General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MFP1

Unit Further Pure 1

Tuesday 17 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The quadratic equation

$$2x^2 + 7x + 8 = 0$$

has roots α and β.

(a) Write down the values of $\alpha + \beta$ and $\alpha\beta$. *(2 marks)*

(b) Show that $\alpha^2 + \beta^2 = \frac{17}{4}$. *(2 marks)*

(c) Find a quadratic equation, with integer coefficients, which has roots

$$\frac{1}{\alpha^2} \text{ and } \frac{1}{\beta^2}$$ *(5 marks)*

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2 Show that only one of the following improper integrals has a finite value, and find that value:

(a) $\int_8^\infty x^{-\frac{2}{3}}\,dx$;

(b) $\int_8^\infty x^{-\frac{4}{3}}\,dx$. *(5 marks)*

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3 (a) Solve the following equations, giving each root in the form $a + bi$:

(i) $x^2 + 9 = 0$; *(1 mark)*

(ii) $(x + 2)^2 + 9 = 0$. *(1 mark)*

(b) (i) Expand $(1 + x)^3$. *(1 mark)*

(ii) Express $(1 + 2i)^3$ in the form $a + bi$. *(3 marks)*

(iii) Given that $z = 1 + 2i$, find the value of

$$z^* - z^3$$ *(2 marks)*

4 (a) Use the formulae for $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r^2(4r-3) = kn(n+1)(2n^2-1)$$

where k is a constant. *(5 marks)*

(b) Hence evaluate

$$\sum_{r=20}^{40} r^2(4r-3)$$ *(2 marks)*

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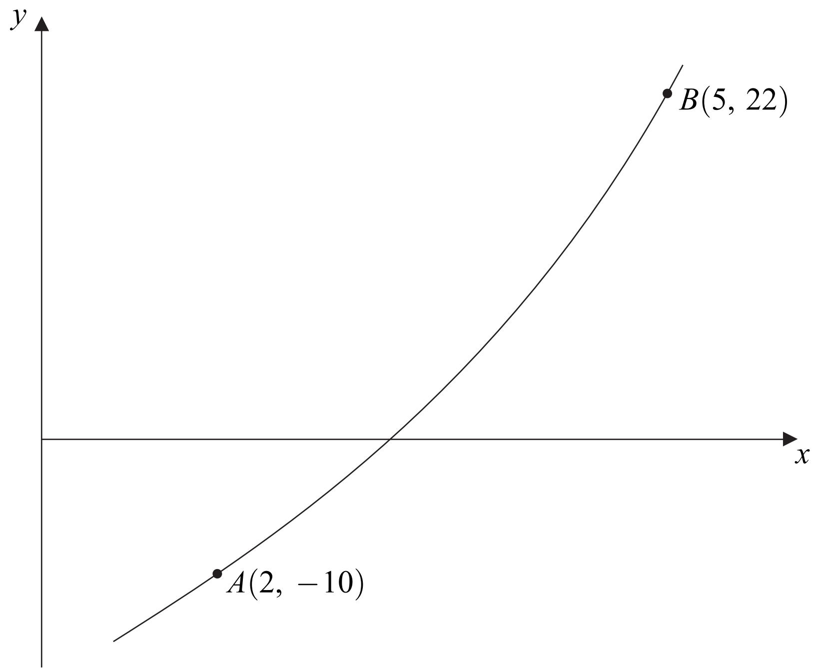
5 The diagram below (not to scale) shows a part of a curve $y = \mathrm{f}(x)$ which passes through the points $A(2, -10)$ and $B(5, 22)$.

(a) (i) On the diagram, draw a line which illustrates the method of linear interpolation for solving the equation $\mathrm{f}(x) = 0$. The point of intersection of this line with the x-axis should be labelled P. *(1 mark)*

(ii) Calculate the x-coordinate of P. Give your answer to one decimal place. *(3 marks)*

(b) (i) On the same diagram, draw a line which illustrates the Newton–Raphson method for solving the equation $\mathrm{f}(x) = 0$, with initial value $x_1 = 2$. The point of intersection of this line with the x-axis should be labelled Q. *(1 mark)*

(ii) Given that the gradient of the curve at A is 8, calculate the x-coordinate of Q. Give your answer as an exact decimal. *(2 marks)*



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6 Find the general solution of each of the following equations:

(a) $\tan\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right) = \dfrac{1}{\sqrt{3}}$; *(4 marks)*

(b) $\tan^2\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right) = \dfrac{1}{3}$. *(3 marks)*

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7 A hyperbola H has equation

$$\frac{x^2}{9} - y^2 = 1$$

(a) Find the equations of the asymptotes of H. *(1 mark)*

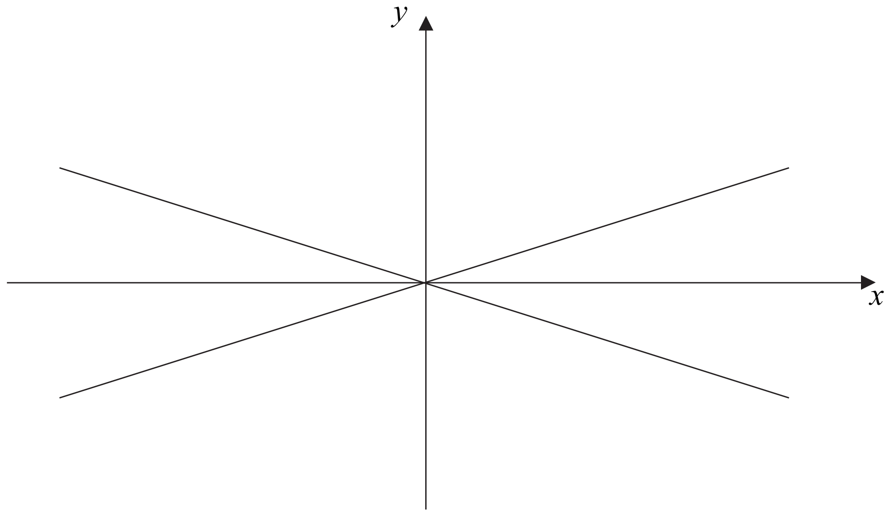
(b) The asymptotes of H are shown in the diagram opposite. On the same diagram, sketch the hyperbola H. Indicate on your sketch the coordinates of the points of intersection of H with the coordinate axes. *(3 marks)*

(c) The hyperbola H is now translated by the vector $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$.

(i) Write down the equation of the translated curve. *(2 marks)*

(ii) Calculate the coordinates of the two points of intersection of the translated curve with the line $y = x$. *(4 marks)*

(d) From your answers to part **(c)(ii)**, **deduce** the coordinates of the points of intersection of the original hyperbola H with the line $y = x - 3$. *(2 marks)*



8 The diagram below shows a rectangle R_1 which has vertices (0, 0), (3, 0), (3, 2) and (0, 2).

(a) On the diagram, draw:

(i) the image R_2 of R_1 under a rotation through 90° clockwise about the origin; *(1 mark)*

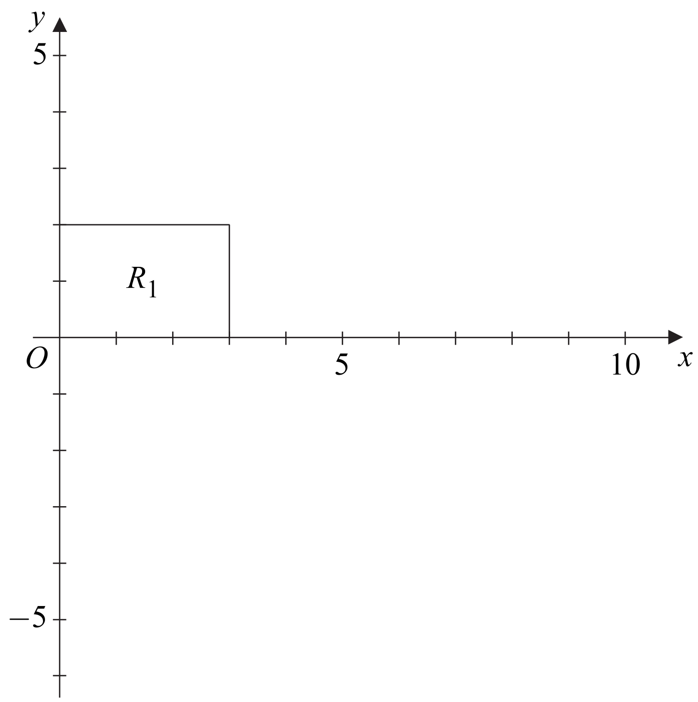
(ii) the image R_3 of R_2 under the transformation which has matrix

$$\begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$ *(3 marks)*

(b) Find the matrix of:

(i) the rotation which maps R_1 onto R_2; *(1 mark)*

(ii) the combined transformation which maps R_1 onto R_3. *(3 marks)*



9 A curve has equation

$$y = \frac{x}{x-1}$$

(a) Find the equations of the asymptotes of this curve. *(2 marks)*

(b) Given that the line $y = -4x + c$ intersects the curve, show that the x-coordinates of the points of intersection must satisfy the equation

$$4x^2 - (c+3)x + c = 0$$ *(3 marks)*

(c) It is given that the line $y = -4x + c$ is a tangent to the curve.

(i) Find the two possible values of c.

(No credit will be given for methods involving differentiation.) *(3 marks)*

(ii) For each of the two values found in part **(c)(i)**, find the coordinates of the point where the line touches the curve. *(4 marks)*